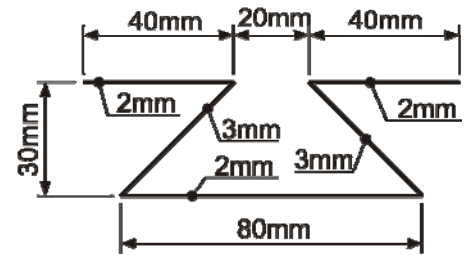


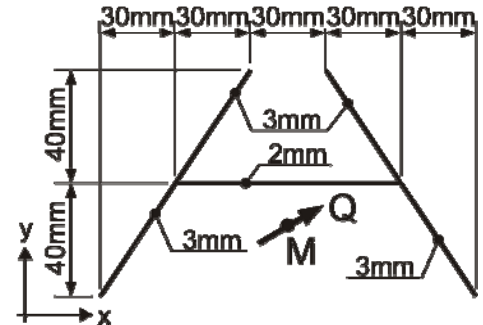
1. Feladat (25 pont):

- Határozza meg a vázolt szelvény M nyírási középpontjának helyét!
- Határozza meg a keresztmetszet ω^* redukált cikkterület függvényét a jellemző értékek feltüntetésével!

**2. Feladat (25 pont):**

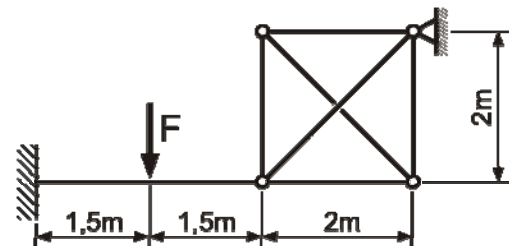
- Határozza meg a vázolt nyílt vékonyfalú keresztmetszet M nyírási középpontján átmenő Q nyíróerő hatására kialakuló τ feszültség eloszlását (és irányát) a jellemző értékek feltüntetésével!

Adatok: $Q_x = 8000 \text{ N}$; $Q_y = 6000 \text{ N}$

**3. Feladat (25 pont):** A vázolt vegyes szerkezetet egy darab koncentrált F erő terheli.

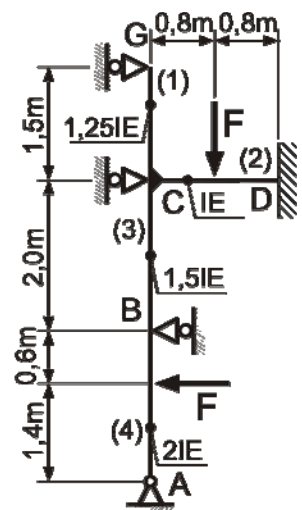
- Erőmódszerrel határozza meg a gerenda nyomatéki ábráját és a rácsrudak normálerő igénybevételeit a jellemző értékek feltüntetésével!
- Határozza meg az F erő támadáspontjának erő irányú elmozdulását!

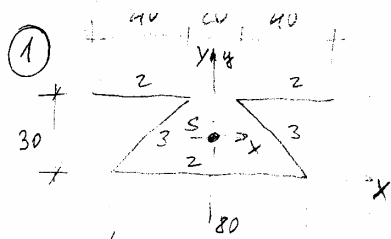
Adatok: $F = 4000 \text{ N}$; $IE = 1,5 \cdot 10^{10} \text{ Nmm}^2$; $AE = 3 \cdot 10^7 \text{ N}$

**4. Feladat (25 pont):** A vázolt gerenda szerkezet elemeinek húzó-nyomó merevsége végtelen.

- Mozgásmódszerrel határozza meg a gerenda elemek hajlító nyomatéki ábráit a jellemző értékek feltüntetésével!

Adatok: $F = 16 \text{ kN}$; $AE = \infty$

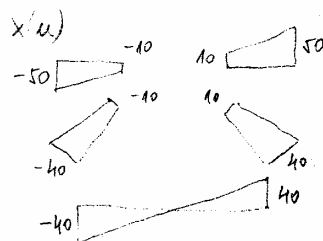




$$A = 160 \cdot 2 + 30 \cdot \sqrt{2} \cdot 3 \cdot 2 = 320 + 254,56 = 574,56 \text{ mm}^2$$

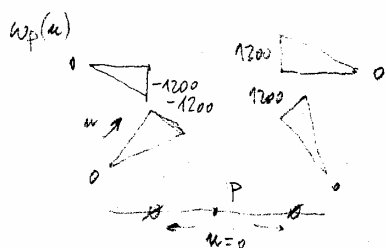
$$S_x = 80 \cdot 2 \cdot 30 + 2 \cdot 15 \cdot 3 \cdot 30 \sqrt{2} = 4800 + 3818,38 = 8618,38 \text{ mm}^3$$

$$Y_S = \frac{S_x}{A} = 15 \text{ mm}$$



$$J_x = 2 \cdot 40 \cdot 2 \cdot 15^2 + 2 \cdot 3 \cdot 30 \sqrt{2} \cdot \frac{30^3}{12} + 2 \cdot 80 \cdot 15^2 = 36000 + 19091,88 + 3600 = 91091,88 \text{ mm}^4$$

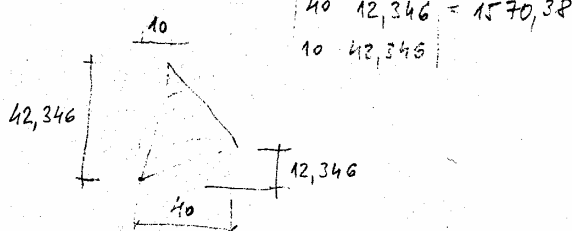
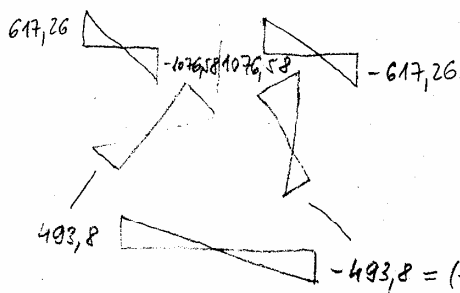
$$J_y = 2 \cdot 40 \cdot 2 \cdot \left(30^2 + 2 \cdot 3 \cdot 30 \sqrt{2} \cdot \left(25^2 + \frac{30^2}{12} \right) + 2 \cdot 80 \cdot \frac{80^2}{12} \right) + 172190,91 + 85333,3 = 428857,58$$



$$\int x(u) w_p(u) u(u) du = 2 \cdot \left\{ 2 \cdot \frac{40}{6} \left[2 \cdot 1200 \cdot 10 + 1200 \cdot 50 \right] + 3 \cdot \frac{30 \sqrt{2}}{6} \left[2 \cdot 1200 \cdot 40 + 40 \cdot 1200 \right] \right\} = 2 \cdot (1120000 + 1527350,65) = 5294701,3$$

$$\eta_M = - \frac{5294701,3}{428857,58} = -12,346 \text{ mm}$$

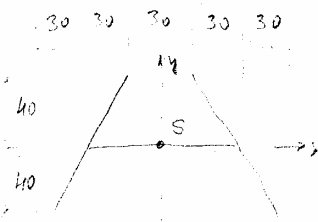
$$\omega_0 = \omega^* \quad M(0, -12,346)$$



$$-493,8 = (-12,346) \cdot 40$$

② $T_x = -\frac{Q_y S_{xy}}{I_y}$

$T_y = -\frac{Q_x S_{yx}}{I_x}$

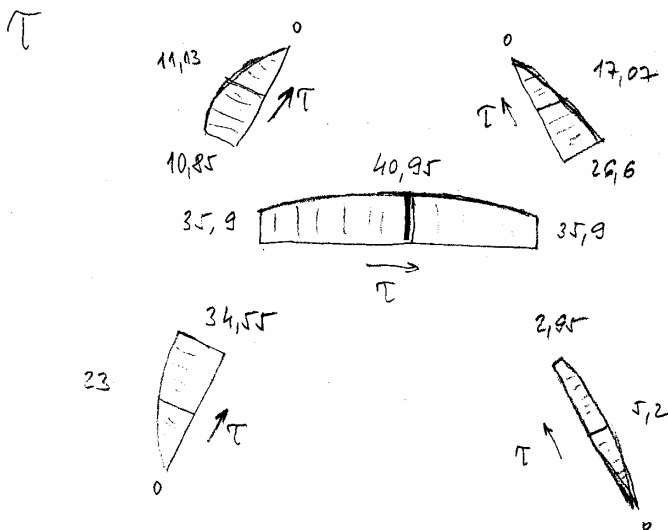
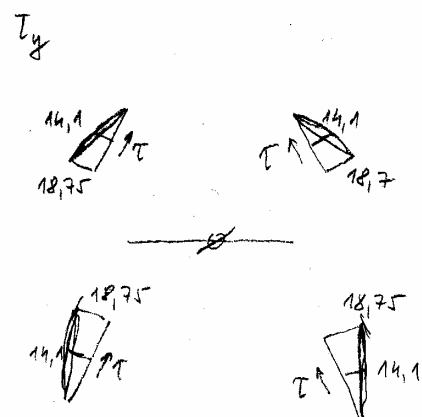
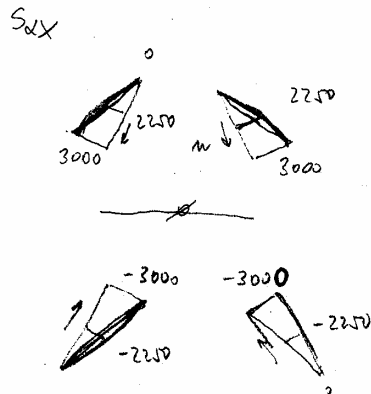
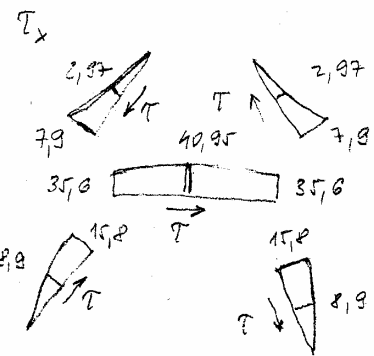
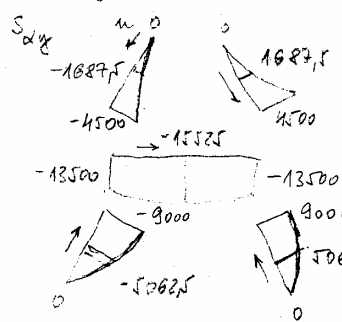
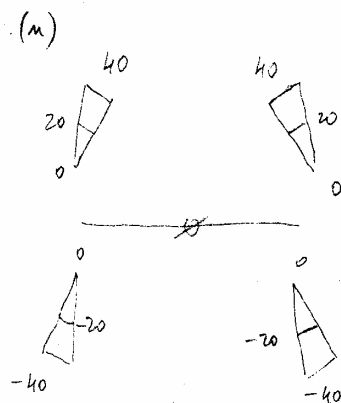
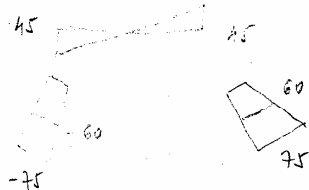
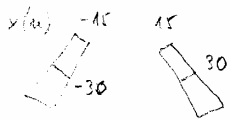


$$I_x = 4 \cdot 50 \cdot 3 \left(20^2 + \frac{40^2}{12} \right) = 320\,000 \text{ mm}^4$$

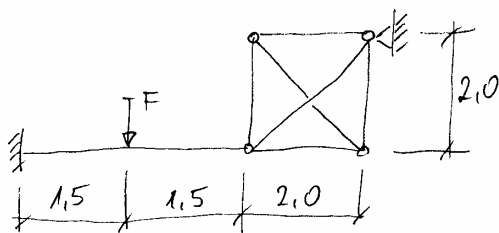
$$I_y = 90 \cdot 2 \cdot \frac{90^2}{12} + 2 \cdot \left[50 \cdot 3 \left(30^2 + \frac{30^2}{12} \right) + 50 \cdot 3 \cdot \left(60^2 + \frac{30^2}{12} \right) \right] = 121\,500 + 300 \cdot (975 + 3675) = 1516\,500 \text{ mm}^4$$

$$-\frac{Q_x}{Q_y} = -0,0052753$$

$$-\frac{Q_y}{I_x} = -0,01875$$



③

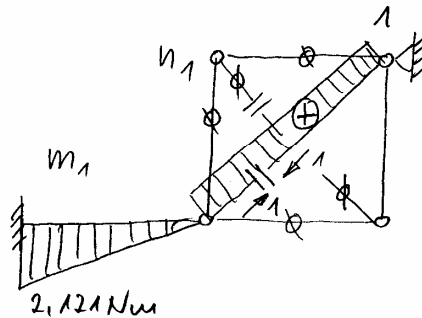
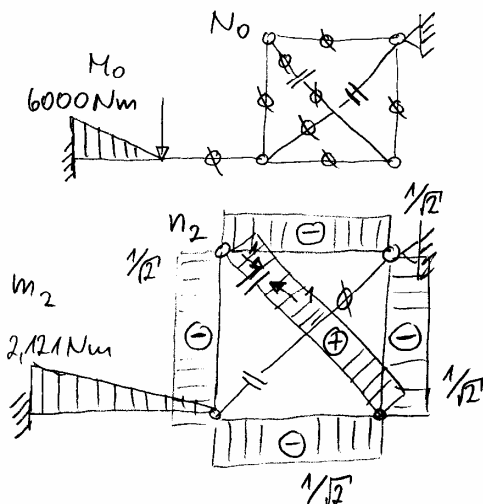


$$F = 4000 \text{ N}$$

$$1E = 1,5 \cdot 10^{10} \text{ Nmm}^2 = 1,5 \cdot 10^4 \text{ Nm}^2$$

$$AE = 3 \cdot 10^7 \text{ N}$$

2x határozatlan!



$$\delta_{10}^N = \frac{1}{1E} \left[\frac{6000 \cdot 1,5}{2} \cdot \frac{5}{6} \cdot -2,121 \right] = \frac{-7953,75}{1E} = -0,53025 \text{ m}$$

$$\delta_{20}^N = \frac{1}{1E} \left[\frac{6000 \cdot 1,5}{2} \cdot \frac{5}{6} \cdot 2,121 \right] = \frac{7953,75}{1E} = 0,53025 \text{ m}$$

$$\delta_{11}^N = \frac{1}{1E} \left[\frac{2,121 \cdot 3}{2} \cdot \frac{2}{3} \cdot 2,121 \right] + \frac{1}{AE} [1 \cdot 2\sqrt{2} \cdot 1] = \frac{4,4986}{1E} + \frac{2,8284}{AE} = 3 \cdot 10^{-4} \text{ m}$$

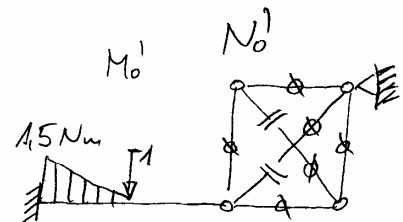
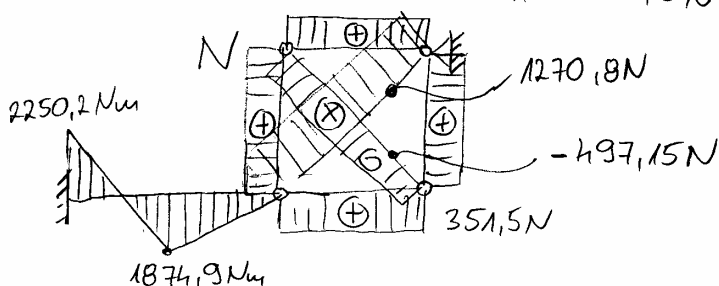
$$\delta_{12}^N = \delta_{21}^N = \frac{1}{1E} \left[\frac{2,121 \cdot 3}{2} \cdot \frac{2}{3} \cdot -2,121 \right] + \frac{1}{AE} [0] = -2,9991 \cdot 10^{-4} \text{ m}$$

$$\delta_{22}^N = \frac{1}{1E} \left[\frac{2,121 \cdot 3}{2} \cdot \frac{2}{3} \cdot 2,121 \right] + \frac{1}{AE} [1 \cdot 2\sqrt{2} \cdot 1 + 4 \cdot \frac{1}{\sqrt{2}} \cdot 2 \cdot \frac{1}{\sqrt{2}}] = \frac{4,4986}{1E} + \frac{6,8284}{AE} = 3,0014 \cdot 10^{-4} \text{ m}$$

$$\begin{cases} 3 \cdot x_1 - 2,9991 \cdot x_2 = 0,53025 \cdot 10^4 \\ -2,9991 x_1 + 3,0014 \cdot x_2 = -0,53025 \cdot 10^4 \end{cases} \quad \begin{cases} x_1 = 1270,8 \text{ N} \\ x_2 = -497,15 \text{ N} \end{cases}$$

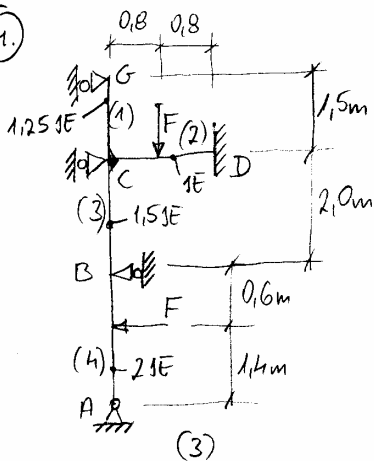
$$-2,9991 \cdot x_1 + 3,0014 \cdot \left(\frac{3 \cdot x_1 - 5302,5}{2,9991} \right) = -5302,5$$

$$0,0032 x_1 = 4,06647 \rightarrow x_1 = 1270,8 \text{ N}$$



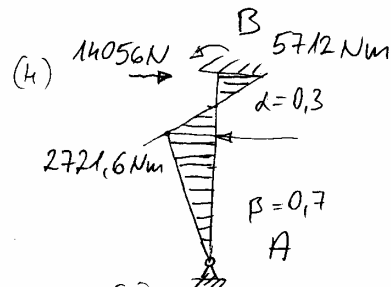
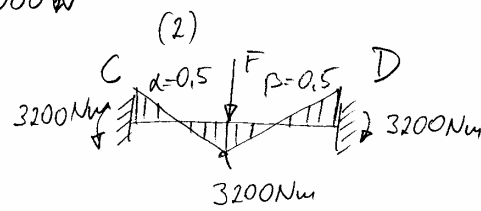
$$e_F = \int \frac{M \cdot M_0'}{1E} ds + \int \frac{N \cdot N_0'}{AE} ds = \frac{1}{1E} \left[\frac{1,5 \cdot 1,5}{2} \cdot 875,17 \right] = \frac{984,56}{1E} = 0,0656 \text{ m}$$

(4.)



$$F = 16\,000\text{ N}$$

$$EI = \infty$$



$$B: \frac{4(1,5EI)}{2} = 3EI$$

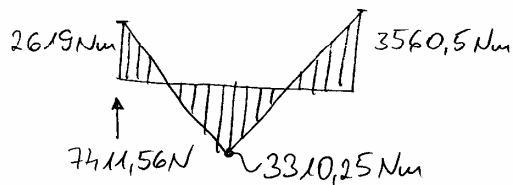
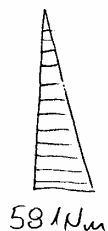
$$\frac{3(2EI)}{2} = 3EI$$

$$C: \frac{3(1,25EI)}{1,5} = 2,5EI$$

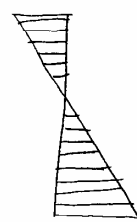
$$\frac{4(1EI)}{1,6} = 2,5EI$$

$$\frac{4(1,5EI)}{2} = 3EI$$

D	C			B	
2	2	1	3	3	4
1	$\frac{5}{16} = 0,3125$	$\frac{5}{16} = 0,3125$	$\frac{6}{16} = 0,375$	$\frac{1}{2}$	$\frac{1}{2}$
+3200	-3200				-5712
+500	+1000	+1000	+1200	+600	
			+1278	+2556	+2556
-129,69	-399,375	-399,375	-479,25	-239,625	
			+59,91	+119,81	+119,81
-9,36	-18,72	-18,72	-22,46	-11,23	
			+2,81	+5,62	+5,62
-0,44	-0,88	-0,88	-1,05	-0,53	
				+0,26	+0,26
3560,5	-2619	+581	+2038	+3030,3	-3030,3



2038 N/m



3030,3 N/m

3030,3 N/m

